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# Hamiltonian chaos and localization in magnetic multilayer system

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**Abstract**

We investigated dynamical effects in a system of magnetic multilayers which arise at different rates of change of the external field. Our studies have been stimulated by recent experiments indicating nontrivial magnetic dynamics in these systems.

To describe a system consisting of magnetic layers, we have used conventional Bloch equations. A time-dependent external magnetic field has been applied, but no dissipation effects have been taken into account. We have studied the time development and instabilities of the regular behaviour of this system numerically. We found local energy excitations (breathers) and chaotic transients. The behaviour and the final configurations can strongly depend on the initial conditions, and the strength of the external field at an earlier time. We observed some sudden switching between two remarkably different states. The series of bifurcations has been found.

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Magnetic multilayer systems are considered as one of the most promising devices with many useful properties such as giant magnetoresistance, spin-wave transistors and spin-effect amplifiers. Therefore, they have attracted considerable attention in recent years. These magnetic layers can be produced from Fe, Co or Ni separated by non-magnetic dielectric or metallic layers. They are built by modern growth techniques such as laser ablation or molecular bean epitaxy (MBE). In several cases the exchange interaction between moments in one layer is much stronger than between moments in different layer. In these cases all moments in one particular layer have the same orientation. Therefore, the layer behaves like a single domain, so we can treat the system as a one-dimensional spin-chain with magnetization vector $\mathbf{M}^{(n)}$ of the $n$th layer. We have taken into account anisotropy stronger than the magnetic interlayer-interaction, because strong interactions destroy all stable configurations immediately. To describe the system we used the following Hamiltonian (1):

$$E = -J\sum_{n=1}^{N}\mathbf{M}^{(n)}\mathbf{M}^{(n+1)} - \frac{b}{2}\sum_{n=1}^{N}\left(M_z^{(n)}\right)^2 - \sum_{n=1}^{N}\mathbf{M}^{(n)}\mathbf{H}^{(n)} \qquad (1)$$

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where $J$ is the exchange-energy term, $b$ is the anisotropy coefficient and $\mathbf{H}^{(n)}$ is the external field applied to the $n$th layer.

The phenomenon that nonlinearity and discreteness in lattices can lead to spatially localized stable modes was discovered quite a few years ago. These modes were named breathers and rotobreathers. There have been some intensive investigations of discrete breathers [2], especially in spin lattices [3, 4]. However, there are still many open questions about them, both theoretical and experimental. For example, we do not know much about how to create and how to destroy them. In order to get more information about this, we studied the time evolution of breathers and other kinds of localization in magnetic mono- and multilayers.

From the energy-formula above and using the Landau–Lifshitz equations

$$\mathbf{h}_{\text{eff}}^{(n)} = -\frac{\mathrm{d}E}{\mathrm{d}\mathbf{M}^{(n)}} \tag{2}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{M}^{(n)} = \text{const}\,\mathbf{M}^{(n)} \times \mathbf{h}_{\text{eff}}^{(n)} \tag{3}$$

we obtain the dynamical equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{M}^{(n)} = J\mathbf{M}^{(n)} \times (\mathbf{M}^{(n+1)} + \mathbf{M}^{(n-1)}) + b\mathbf{M}^{(n)} \times \mathbf{e_z}(\mathbf{M}^{(n)}\,\mathbf{e_z}) + \mathbf{M}^{(n)} \times \mathbf{H}^{(n)}. \tag{4}$$

If we introduce a polar coordinate system, where $\mathbf{M}^{(i)} = M^{(i)}(\cos\phi^{(i)}\sin\theta^{(i)}, \sin\phi^{(i)}\sin\theta^{(i)}, \cos\theta^{(i)})$, and consider the magnitude of the magnetic vectors constant ($M^{(i)} = M = 1$), we get the following differential-equation system:

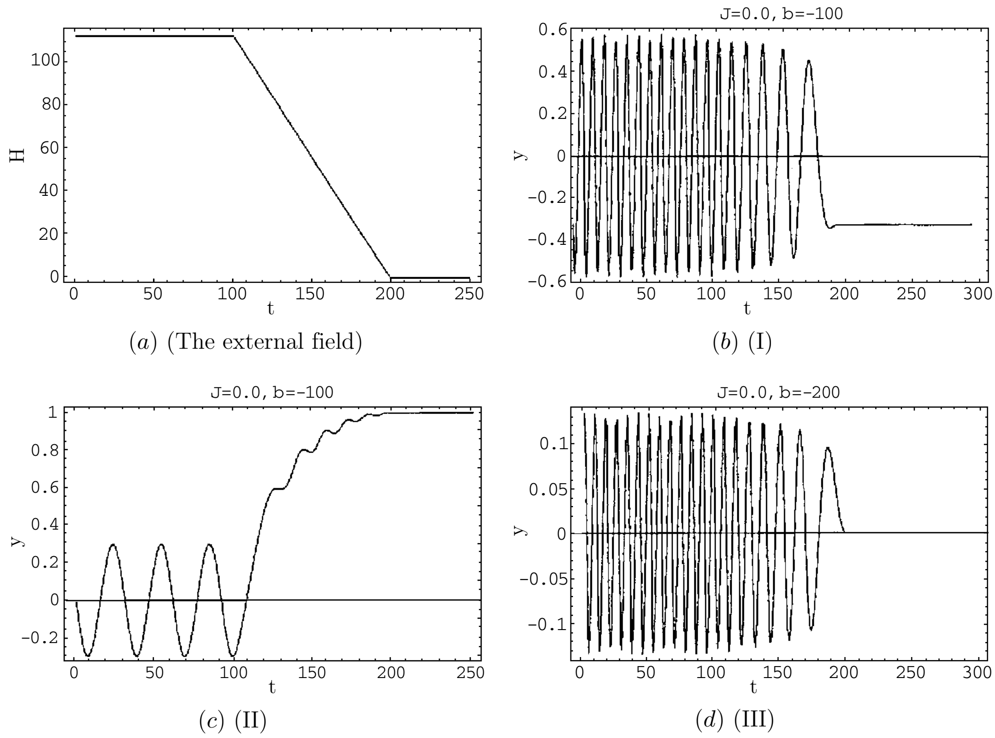
$$\begin{aligned}\frac{\mathrm{d}\theta^{(n)}}{\mathrm{d}t} &= J[\sin\theta^{(n-1)}\sin(\phi^{(n)} - \phi^{(n-1)}) + \sin\theta^{(n+1)}\sin(\phi^{(n)} - \phi^{(n+1)})] \\ &\quad + \sin\phi^{(n)}H_x^{(n)} - \cos\phi^{(n)}H_y^{(n)} \\ \frac{\mathrm{d}\phi^{(n)}}{\mathrm{d}t} &= J[\cos\theta^{(n)}(\sin\theta^{(n-1)}\cos(\phi^{(n)} - \phi^{(n-1)}) \\ &\quad + \sin\theta^{(n+1)}\cos(\phi^{(n)} - \phi^{(n+1)}))/\sin\theta^{(n)} - (\cos\theta^{(n-1)} + \cos\theta^{(n+1)})] \\ &\quad + b\cos(\theta^{(n)}) - H_z^{(n)} + \left(\sin\phi^{(n)}H_y^{(n)} + \cos\phi^{(n)}H_x^{(n)}\right)\cos\theta^{(n)}/\sin\theta^{(n)}\end{aligned} \tag{5}$$

where $H_x^{(n)}$, $H_y^{(n)}$ and $H_z^{(n)}$ are the components of the applied external field, which is in the $x-y$ plane now $\left(H_z^{(n)} = 0\right)$, moreover $H_x^{(n)} = -2 \times H_y^{(n)}$, and the boundary conditions were open ($\mathbf{M}^{(0)} = 0$, $\mathbf{M}^{(N+1)} = 0$). The hard-axis anisotropy was high ($b = -100$), therefore the main energy term is $\frac{-b}{2}\sum_n \cos^2(\theta^{(n)})$, so we will concentrate on $\cos(\theta^{(i)})$. We chose these values of $b$ and $H_i^{(n)}$ to get stable breathers easily. We use the following indicator to study chaotic transients: we change $\theta^{(i)}$ by a small number (and denote it by $\tilde{\theta}^{(i)}$), than calculate the logarithm of the deviation of the modified $\theta^{(i)}$ from the original value of $\theta^{(i)}$ divided by the initial deviation $d(t) := \ln\left(\left|\frac{\theta^{(i)}(t) - \tilde{\theta}^{(i)}(t)}{\theta^{(i)}(0) - \tilde{\theta}^{(i)}(0)}\right|\right)$, than examine the first few values (i.e., at the first few numerical timesteps) of this function. In order to understand the multilayer system, we first study the behaviour of one layer, which is, however, nontrivial, then mention a few facts about the multilayer case.

Consider a system consisting of one layer only (it means that we ignore the term with $J$), therefore the dynamical equations become

$$\begin{aligned}\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \sin(\phi)H_x - \cos(\phi)H_y \\ \frac{\mathrm{d}\phi}{\mathrm{d}t} &= b\cos(\theta) + (\sin(\phi)H_y + \cos(\phi)H_x)\cos(\theta)/\sin(\theta).\end{aligned} \tag{6}$$

If the external field is constant, the behaviour is totally regular, the Fourier spectra show only one frequency.




($a$) (The external field) ($b$) (I) ($c$) (II) ($d$) (III)

**Figure 1.** Different behaviour of $y = \cos(\theta)$, $H = \sqrt{H_x^2 + H_y^2 + H_z^2}$ is the absolute value of the external field, $H$ changes only between $t_1$ and $t_2$, here $t_1 = 100$ and $t_2 = 200$.

**Table 1.**

| $H_{\mathrm{ini},y}$ | $\cos(\theta)$ if fast | $\cos(\theta)$ if normal | $\cos(\theta)$ if slow |
|---|---|---|---|
| 50 | 0.999 4 | −0.9931 | 0.9913 |
| 49 | −0.985 89 | −0.9790 | −0.9923 |
| 45 | −0.962 7 | 0.9892 | 0.9713 |
| 40 | −0.964 4 | −0.9737 | 0.9861 |
| 35 | 0.895 6 | −0.9377 | 0.9409 |
| 20 | 0.790 3 | 0.7852 | 0.7808 |

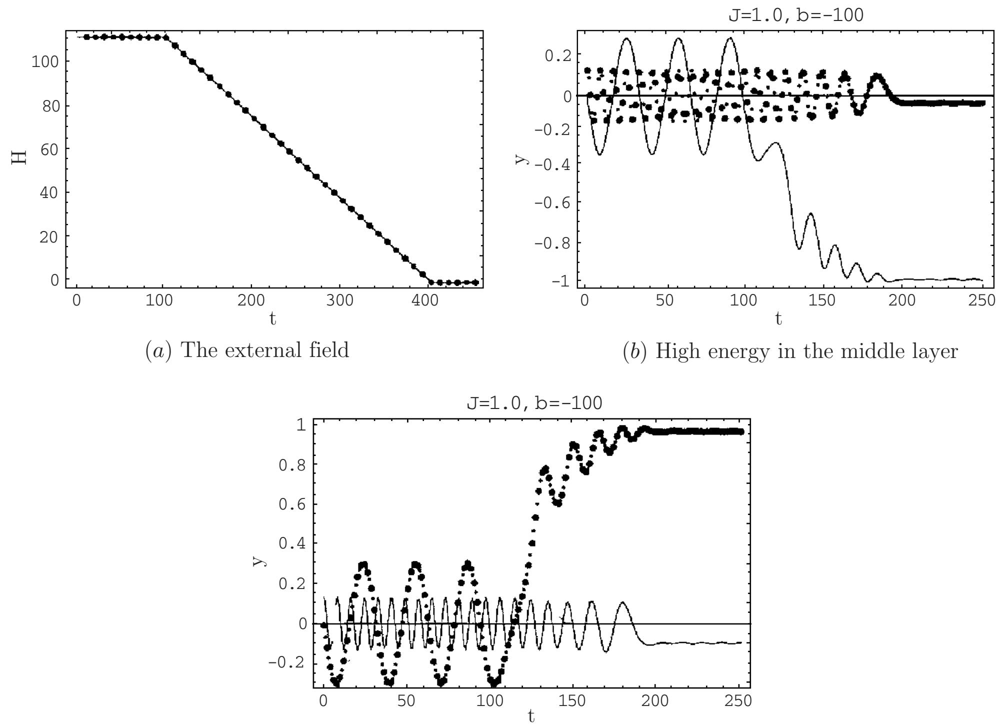
Dependence of the final $\cos(\theta)$ on the initial field if the rate of change is fast ($t_2 - t_1 = 100$), normal ($t_2 - t_1 = 300$) or slow ($t_2 - t_1 = 1000$) at the behaviour-type II.

If we reduce the field continuously to 0, we can observe different kinds of behaviour (figure 1). (I) is quite typical behaviour, we can find similar in the case of a wide range of initial conditions, but not for cases (II) and (III).

In case (II), the $t = 0$ angles $\theta_{\mathrm{ini}} = 1.522\,314$, $\phi_{\mathrm{ini}} = -0.357\,3936$ and after time $t_2$ the M vector is very close to the hard $z$-axis: $\cos(\theta_{\mathrm{fin}})$ is 0.999 49. In table 1, the dependence of the final $\cos(\theta)$ on the initial field with different rates of change of the field can be seen.

In case (III), $\theta_{\mathrm{ini}} = -1.473\,89$, $\phi_{\mathrm{ini}} = -0.334\,493\,19$.

So when there is no coupling and the field decreases to zero, the final $\cos(\theta)$ is constant, and the dependence of this constant on the initial angle of the magnetization vector is quite interesting. The sign of $\cos(\theta)$ can change very suddenly. According to it we can divide the two-dimensional initial space $(\theta, \phi)$ into two disjoint domains with sharp boundaries, one domain results positive, the other negative $\cos(\theta_{\mathrm{fin}})$. For example, if $(\theta, \phi)$




(*a*) The external field

(*b*) High energy in the middle layer

(*c*) Low energy in the middle layer

**Figure 2.** Creating breathers by setting special initial conditions; $\cos(\theta^{(2)})$ is the continuous line, the third layer (large dots) is the same as the first one (small dots), the external field is uniform in space.

is $(0.541\,823 \times \pi/2, 0.2)$, $\cos(\theta_{\text{fin}})$ is about $-0.7$, and if $(\theta, \phi) = (0.541\,822 \times \pi/2, 0.2)$, $\cos(\theta_{\text{fin}}) = 0.67$.

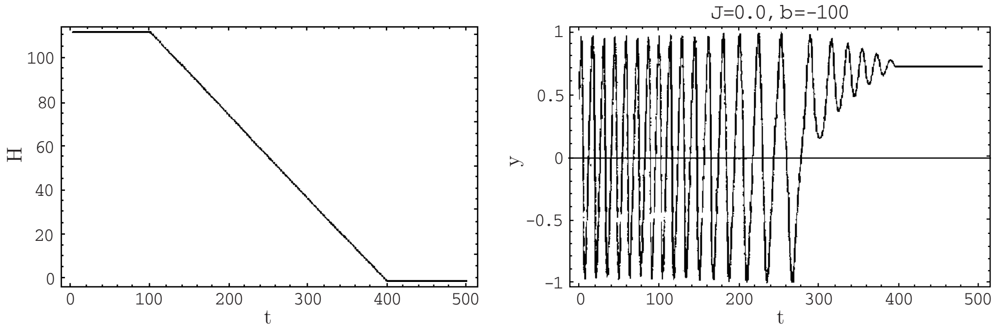
Combining cases (II) and (III) we can create breathers (local excitations). Let us consider a three-layer system ($J \neq 0$) with the following initial conditions: the first and third layers are the same as in case (III) and the middle layer is the same as in case (II) (figure 2(*b*)) or vice versa (figure 2(*c*)).

There is another typical behaviour (figure 3).

If the final value of the external field $H_{\text{fin}}$ is not 0 but less than the initial field was (for example $H_{\text{fin}} = 0.4 \times H_{\text{ini}}$), the magnetization after $H$ reaches $H_{\text{fin}}$ will not be constant but oscillate with smaller frequency, and if $H_{\text{fin}}$ is even smaller (for example, $H_{\text{fin}} = 0.2 \times H_{\text{ini}}$), also with a smaller amplitude. There is a critical point here, if $H_{\text{fin}} = 0.360\,417\,8 \times H_{\text{ini}}$, the amplitude of $\cos(\theta)$ drops, if $H_{\text{fin}} = 0.360\,418 \times H_{\text{ini}}$, it does not drop (figure 4).

The local instability indicator can be positive when $H$ reaches the critical point (figure 5) where the time units are taken in numerical timesteps, time 0 was 399 above, on the vertical axis the indicator introduced above.

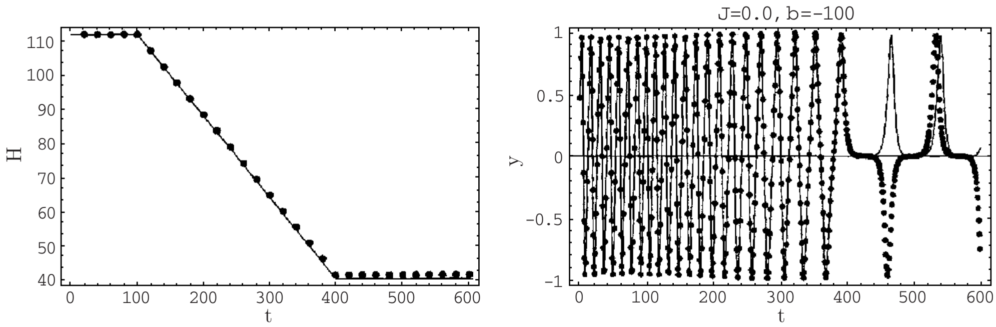
When we tried to find the dependence critical field from the initial field, we found a very unexpected phenomenon: Sometimes there is not only one critical field, but many, i.e. changing the final field for a fixed initial field we can find regions with high- and low-amplitude of the final $\cos(\theta)$, alternately, and the boundaries between these regions are very sharp. For example, if the initial conditions are $\theta_{\text{ini}} = 1.5$, $\phi_{\text{ini}} = -1.3$, $t_2 - t_1 = 300$, then the final




$(a)$ The external field

$(b)$ (IV)

**Figure 3.** Other typical behaviour of $y = \cos(\theta)$, $H$ is the absolute value of the external field and changes only between $t_1$ and $t_2$; here $t_1 = 100$ and $t_2 = 400$, we can observe that at time about $t_1 = 300$ the amplitude of $y$ suddenly drops.




$(a)$ The external field

$(b)$ Different behaviour of $y = \cos(\boldsymbol{\theta})$

**Figure 4.** Sharp boundary between two different kinds of behaviour, $H_{\mathrm{fin}} = 0.36$ (continuous line) and $H_{\mathrm{fin}} = 0.361$ (dotted line), but the behaviour is remarkably different.

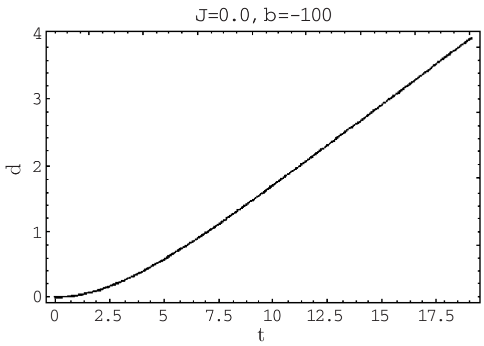



**Figure 5.** The instability indicator seems to be positive, $t = 0$ at about the critical point.

amplitude will be about $1/2$ for $H_{\mathrm{fin}} < 19.8866$ but almost one for $19.8867 < H_{\mathrm{fin}} < 20.4422$, less than $1/2$ again for $20.4423 < H_{\mathrm{fin}} < 21.4178$ and almost one again for $H_{\mathrm{fin}} > 21.4179$. If we examine it in the case of a higher initial field, we can find more regions. In figure 6, one little line means that if the initial field is the given $H_i$ then if we reduce the field to a value a

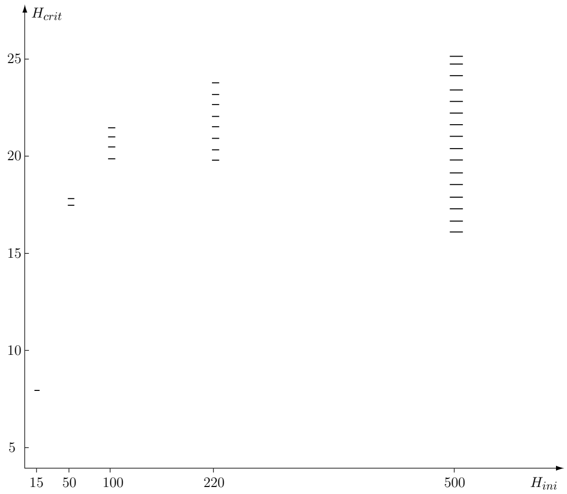
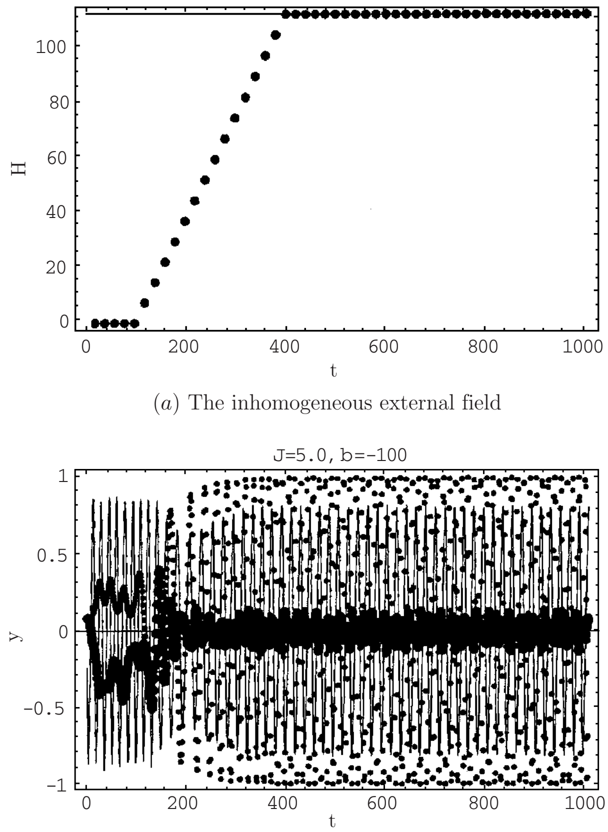



**Figure 6.** The short lines represent boundaries between remarkably different regions of the dynamical behaviour of the single-layer magnitization at a given initial field $H_{\mathrm{ini}}$.




($a$) The inhomogeneous external field

($b$) $y = \cos(\boldsymbol{\theta})$

**Figure 7.** Damping in the second layer—the continuous line is $\cos(\theta^{(1)})$, the thick dotted is $\cos(\theta^{(2)})$ and the thin dotted is $\cos(\theta^{(3)})$.

little bit greater than the $H_{\mathrm{crit}}$ coordinate of the line, the amplitude will be close to one but if we decrease it slightly below $H_{\mathrm{crit}}$, the amplitude drops close to $1/2$.

If the coupling is stronger, we can observe a different kind of localization (figure 7). In this figure, we can see that the amplitude of $\cos(\theta^{(2)})$ is much smaller than the amplitudes of $\cos(\theta^{(1)})$ and $\cos(\theta^{(3)})$, and it is small for a very long time, so this damping is a stable phenomenon although the coupling is not very small.

## Summary

We studied magnetic layers with weak or zero coupling and huge anisotropy and applied a changing inhomogeneous external field to them. We found that even the zero-coupling case is highly nontrivial, there are some chaotic transients between different states, strong sensitivity from the initial conditions and series of bifurcations.

If the coupling is nonzero but weak, we can produce almost arbitrary local energy excitations (breathers) and other kinds of localization by inhomogeneity in the initial magnetizations but not in the external field and in the anisotropy.

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